Exercise 1.11

The ratio is trivial: let $\kappa = X/Y$. If X is scaled by λ to $X' = \lambda X$, then Y is scaled to $Y' = \lambda Y$. This gives us the ratio:

$$\kappa' = \frac{X'}{Y'} = \frac{\lambda X}{\lambda Y} = \frac{X}{Y} = \kappa \tag{1}$$

so the ratio κ is unchanged with a scaling in X; *i.e.*, it is intensive.

The derivative takes a little more thought. We define $X(Y)$ as an extensive function of an extensive variable. This means that:

$$\lambda X(Y) = X(\lambda Y) = X(Y') \tag{2}$$

where we have defined $Y' = \lambda Y$.

Now we can take the derivative of left and right sides:

$$\lambda \frac{\partial X}{\partial Y} = \frac{\partial X}{\partial Y'} \frac{\partial Y'}{\partial Y} \tag{3}$$

$$= \frac{\partial X}{\partial Y'} \lambda \tag{4}$$

This gives us

$$\frac{\partial X}{\partial Y} = \frac{\partial X}{\partial Y'} \tag{5}$$

which tells us that the derivative $\frac{\partial X}{\partial Y}$ is intensive.