Exercise 4.9

All we have to do here is connect the equations we've been writing to physical reality, and then plug in numbers (and be careful with unit conversions).

First off, the number $\langle N \rangle$ is the number of free electrons in a given volume V. Since we're told to assume that each atom donates an electron (by which I assume *one* electron) to the conducting gas, we need to turn the mass density of copper into atoms per cubic meter. Yay high school chemistry!

$$\begin{aligned} &\frac{9\text{ g Cu}}{\text{cm}^3} \times \frac{\text{moles Cu}}{63.5463\text{ g Cu}} \times \frac{6.022 \times 10^{23}\text{ atoms Cu}}{\text{moles Cu}} \\ &\times \frac{100^3\text{ cm}^3}{\text{m}^3} \times \frac{1\text{ electrons}}{1\text{ atoms Cu}} = 8.53 \times 10^{28} \frac{\text{electrons}}{\text{m}^3} = \frac{\langle N \rangle}{V} \end{aligned} \tag{1}$$

Next, we need to rearrange things to be useful. First, we solve for k_F in the electron density equation:

$$\langle N \rangle = \frac{2V}{(2\pi)^3} \frac{4}{3} \pi k_F^3 \tag{2}$$

$$k_F = \left(3\pi^2 \frac{\langle N \rangle}{V} \right)^{1/3} \tag{3}$$

Now we plug that result into the equation for the Fermi energy:

$$\mu_0 = \frac{\hbar^2}{2m} k_F^2 = \frac{\hbar^2}{2m} \left(3\pi^2 \frac{\langle N \rangle}{V} \right)^{2/3} \tag{4}$$

Finally, let's throw in some numbers:

$$\begin{aligned} \mu_0 / k_\text{B} &= \frac{(1.05 \times 10^{-34}\text{ J}\cdot\text{s})^2}{2 \cdot 9.11 \times 10^{-31}\text{ kg}} \times \left(3\pi^2 \cdot 8.53 \times 10^{28}\text{ m}^{-3}\right)^{2/3} \\ &\quad \times (1.38 \times 10^{-23}\text{ J}\cdot\text{K}^{-1})^{-1} && (5) \\ &= 6.05 \times 10^{-39}\text{ J m}^2 \cdot 9.57 \cdot 1.95 \times 10^{19}\text{ m}^{-2} \cdot 7.25 \times 10^{22}\text{ J}^{-1}\text{ K} && (6) \\ &= 81900\text{ K} && (7) \end{aligned}$$

Hey look, it turns out that Chandler didn't lie to us! This time.[1]

To tie this number to the conclusion Chandler draws, we just have to consider the meaning of the Fermi energy (and the associated Fermi temperature). The Fermi energy is the region where we start to see unoccupied energy states (perhaps a couple of $k_\text{B}T$ below that). So if the temperature associated with that energy (the Fermi temperature) is far above the experimental temperature, the ideal gas term is really all that matters. And, in my experience, 80000 K is far above room temperature.

[1] However, he does write it as $80,000\ ^\circ$K, which is technically incorrect. Kelvin does not take a degree symbol.